Modeling Peak Oil

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Abstract

"Peak oil" refers to the potential future decline in world-wide production of crude oil and its potentially calamitous effects. The vast majority of the literature on peak oil is non-economic and ignores price effects even when analyzing policies. Unfortunately, most of the economic models of depletable resources do not generate production peaks and thus are not applicable. I present four models which generate production peaks in equilibrium. Production increases in the models are driven by: demand increases, cost reductions through advancing technology, cost reductions through reserve additions, and production capacity increases through site development. Production decreases are driven by scarcity. The models do not rely on market failures and thus indicate that a peak in production may well arise from efficient intertemporal optimization. Since the models do not address market failures, they are not suitable for normative analysis. Positive analysis indicates that an energy tax can delay the peak, delay exhaustion, and reduce production volatility. However, an energy tax can have unintended consequences. Depending on whether or not it is anticipated and/or phased in, an energy tax can hasten the peak, hasten exhaustion, and/or increase production volatility.

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1 Introduction

The term "peak oil" has come to be synonymous with a host of concerns about future energy supplies.[1] A vast non-economic literature addresses whether global oil production has peaked or will soon peak; what consequences that could have for fossil fuel dependent societies; and what should be done about it.[2] Unfortunately, most of this literature fails to recognize the role that prices could play in allocating scarce oil. For example, Hirsch *et al.* (2005) notes a wedge between projections of oil production, which peaks, and oil consumption, which does not.[3] The authors analyze a variety of mitigation policies and conclude that prompt action is required to prevent future shortfalls and economic disruptions. However, they do not mention the effects of these policies on prices or the effects of prices on these policies. Similarly, Lovins *et al.* (2005) proposes a variety of demand reduction policies for "getting the United States completely, attractively, and profitably off oil." However, the analysis ignores the fact that these policies would decrease demand for oil, presumably decreasing the price of oil and the profitability of the policies.

Most economic models of depletable resources do not offer additional insight because they do not generate a peak in production.[4] For example, the seminal model of Hotelling (1931) predicts that (net) prices should grow at the rate of interest and that production should steadily decline over time. Extensions of this model for uncertainty, limited capacity, set-up costs, different grades of ore, and increasing costs with cumulative extraction, also do not generate peaks in production.[5] This raises the question as to whether the observed production peak could have arisen from an economic model. Is the production peak itself evidence of some market failure? Is oil peaking just a series of happy (or unhappy) accidents that is not amenable to economic analysis?

This paper answers these questions by presenting four economic models that generate peaks in production without resorting to market failures. The models are solidly based on the

[1] "Peak oil" refers to the peak in U.S. crude oil production in 1970. This peak, also known as Hubbert's peak, was correctly predicted by M.K. Hubbert in 1956.

[2] Recent books include Abdullah (2005), Cooke (2004), Deffeyes (2005), Deffeyes (2001), Kunstler (2005), and Simmons (2005). Numerous websites and on-line discussions (often bordering on hysteria) are devoted to peak oil. In Aug. 2005, the Wall Street Journal hosted an online forum on peak oil in which the invited economists agreed that peak oil is a "greater challenge than the 'looming crisis' in Social Security" and "one of the most important economic transitions that many of us [...will...] witness." Econoblog (2005).

[3] This report, funded by the U.S. Dept. of Energy, is not published and does not appear on the DoE website.

[4] Notable exceptions, discussed below, are Pindyck (1978), Slade (1982) and Livernois and Uhler (1987).

[5] See Krautkraemer (1998) for a survey of this vast literature.

classic Hotelling theoretical framework of optimizing producers and consumers.[6] The resulting equilibrium can then be subjected to a variety of policy initiatives to analyze whether the policies attain their stated objectives and/or have unintended consequences.

The peak in each model is generated by opposing forces tending to increase or decrease equilibrium production. In the models, increasing demand, improvements in technology, additional reserves, and new site development tend to increase production while scarcity tends to decrease production. Each model is presented in Section 2 and evaluated based on existing empirical evidence.

In Section 3, the models are used to analyze a variety of policy proposals. The simulations show that an energy tax can be effective in delaying the peak and exhaustion and in reducing the volatility of production. However, the energy tax can have some surprising, unintended consequences. Depending on whether the tax is anticipated, unannounced, or phased in, the tax can actually hasten the peak, hasten exhaustion, or increase the volatility of production. Section 4 concludes.

2 Models of Peak Oil

This section presents several Hotelling-style models with peaks in production. Unlike the curve fitting model of Hubbert (1956) or the noneconomic literature on peak oil, these models recognize the incentives of both producers and consumers and that these competing effects must be balanced to determine equilibrium prices and consumption. The models are presented in order of increasing complexity and evaluated based on the empirical evidence. While special attention is given to the time series for the U.S. oil industry, the analysis has broader application.

Figure 1 shows that U.S. annual crude production indeed peaked in 1970 at 3.5 billion barrels of oil. Prior to 1970, production increased steadily. After 1970, production generally declined with a slight recovery and peak in 1985. Since 1985, U.S. production has steadily declined.

The first question is whether this peak in production was driven by increases in demand prior to 1970 and decreases in demand thereafter. The 1970's were characterized after all by

[6] The focus is on competitive models. Modeling a monopoly supplier would likely change the shape of the production peak, but not eliminate it.

inefficient cars and uninsulated homes. Figure 2 shows U.S. production and total consumption of crude oil, proxied by U.S. production plus imports. Total crude consumption peaked initially in 1979 at 5.5 billion barrels and then declined rapidly to just under 4.4 billion barrels in 1983—possibly due to the price spike in 1981. After 1983, consumption steadily increased and first surpassed the earlier peak of 5.5 billion barrels in 2001. Thus declines in demand following 1970 likely did not cause U.S. production to fall. However, falling demand is not necessary for a peak in equilibrium production of a depletable resource as the following models show.

2.1 Model 1: Demand shift

In Model 1, the increase in production is caused by increasing demand. Let $D_t(p)$ be aggregate demand derived from the consumer's optimization problem. Let D_t satisfy the usual requirements of a demand curve, *e.g.,* $D_t' < 0$, and let demand be growing over time, *i.e.,* let $D_t(p) \leq D_{t+1}(p)$ for every p.

Now consider supply of a depletable resource. Assume the finite resource stock, S, can be extracted at cost $C(q)$ where $C' > 0$ and $C'' \geq 0$. Thus profit maximization,

$$\max_{q_t} \sum_{t=1}^{\infty} \beta^t [p_t q_t - C(q_t)],$$

is subject to the stock constraint

$$\sum_{t=1}^{\infty} q_t \leq S \tag{1}$$

where β is the discount factor, and q_t and p_t are production and price in period t. The well-known Hotelling supply relation is seen from the first order condition:

$$p_t = C_q + (1+r)^t \lambda$$

where λ is the shadow value of the stock and the rate of interest, r, is defined by $\beta = 1/(1+r)$. Thus equilibrium in the model is characterized by a net price that grows at the rate of interest.

Figure 3 illustrates how production can increase in this model. The *full marginal cost*—defined as the marginal extraction plus marginal scarcity costs, here $C_q + (1+r)^t\lambda$—is increasing over time due to the increasing scarcity of the depletable resource. This tends to decrease production. However, the demand shift tends to increase consumption over time. In this figure, the demand increase effect is larger, and equilibrium production increases. Under reasonable assumptions, the demand increase will eventually be less than the full marginal cost increase, and equilibrium production decreases. Thus the model can predict a peak in equilibrium production.

Although this model is attractive for reasons of clarity and simplicity, it also has a strong empirical prediction: namely, prices should rise over time, even while production is increasing. Figure 4 shows the time series of domestic first purchase price of U.S. crude oil in 2000 dollars.[7] At the dawn of the U.S. petroleum age, the real price was quite high. After stabilizing around $20 in the 1880's, prices stay in the range of $10 to $20 until the sharp increases of the late 1970's.[8] The series climbs in the 1970's and peaks at $54 in 1981 and then falls again to around $20 where it stayed before climbing again above $30 in 2004. Despite dramatic variation, the price series does not possess an increasing trend using "eyeball" regression techniques and clearly shows long periods of declining prices.[9]

Econometric evidence on the crude oil price series has tested for deterministic trends. Slade (1982), using data from 1870-1978, regresses the price series for several commodities, including petroleum, on linear and quadratic trends and finds evidence for quadratic trends. However, this result may be spurious since she did not test for a unit root. Berck and Roberts (1996) extend Slade's data and cannot reject a unit root leading them to conclude that the commodity prices do not have a deterministic trend. Ahrens & Sharma (1997) reject the unit root hypothesis for some commodities, including petroleum. Lee, List, & Strazicich (2005) reject the unit root hypothesis and find evidence of a quadratic trend in petroleum when allowing for structural breaks in the time series. The plot from their linear trend model shows a declining trend through 1896 which suggests that prices have not been monotonically non-decreasing.[10]

Thus Model 1, although predicting a peak, is probably not a credible model for policy analysis since the predicted price path is not supported by empirical evidence. Models 2-4 all

[7] The series is published by EIA and is similar to the price series developed by Manthy (1978). Following EIA, we use the BLS implicit GDP price deflator to calculate real prices. Since the BLS price deflators only extend through 1929, we use the Balke-Gordon GNP price deflator for earlier years. There remain some issues with linking the two deflators.

[8] From 1880 to 1975, the price was less than $10 in only five years and was greater than $20 for only 12 years. There is likely considerably more variation over shorter time horizons.

[9] Several well-publicized bets over natural resource prices have been resolved in favor of the "optimists."

[10] Their analysis finds structural breaks for petroleum in 1896 and 1971 when allowing for a linear trend and breaks in 1914 and 1926 when allowing for a quadratic trend.

generate U-shaped equilibrium price paths and thus are consistent with the econometric evidence on the price series.

2.2 Model 2: Technological change

This model, based on Slade (1982), simply assumes that costs decrease exogenously over time due to technological change. Let the cost function be $C(q,t)$ where $C_q > 0$, $C_{qq} > 0$, $C_t < 0$ and $C_{qt} < 0$. The producer's optimization is given by:

$$\max_{q_t} \sum_{t=1}^{\infty} \beta^t [p_t q_t - C(q_t, t)]$$

subject to the stock constraint in equation 1. The first order condition is

$$p_t = C_q(q_t, t) + (1+r)^t \lambda.$$

This first order condition implies that the net price again grows at the rate of interest.

Figure 5 illustrates a production increase in Model 2 where price is constant, *i.e.*, $p_t = p_{t+1} = \bar{p}$. The marginal cost curve, C_q, shifts down over time due to technological progress. However, the scarcity cost, $(1+r)^t\lambda$, is increasing over time due to scarcity. Thus production can increase if the decrease in marginal extraction costs outweighs the increase in scarcity costs, as illustrated in Figure 5. Since the growth rate of the scarcity cost is r, the scarcity-cost increase will eventually outweigh the decrease in marginal extraction costs (and production will fall) as long as the growth rate of the decline in marginal costs is less than r.

Figure 6 illustrates a simulation of this model.[11] In the baseline simulation, the peak occurs in year 26 and exhaustion occurs after year 42.[12] Figure 6 also shows that a higher interest rate of 7% leads to an earlier peak and earlier exhaustion.

Thus far, only the supply side of the model has been analyzed. Because of increasing supply, equilibrium in this model can lead to a U-shaped price path even with a stationary demand. With a stationary demand, the peak in production would occur simultaneously with the lowest price. If demand increases over time, the peak in production occurs after the low point on the price path.

[11] Parameters of the baseline simulation are $\bar{p} = \$50$, $C(q,t) = .01q^2/2t$, $S = 2,000,000$, and $r = 0.05$.

[12] Production drops off sharply after the peak due to the assumption of linear marginal cost made flatter by cost reductions.

Figure 7 graphs production against the price path along with a quadratic trend.[13] The trend line suggests that the low point in price may have occurred earlier than the peak in production.

Although the relationship between technological change and costs is appealing theoretically, it is difficult to test empirically and existing evidence does not suggest that technological change reduced costs significantly. Cuddington and Moss (2001) construct a measure of technological diffusions but did not find that it significantly reduced exploration and development costs for crude oil reserves—although they find that it reduced finding costs for nonassociated natural gas.

The technological change model is also somewhat unsatisfactory from a theoretical perspective since the primary driver of interest, technological change, is unexplained in the model. The next model does not rely on some exogenous mechanism but has reductions in extraction costs derived endogenously through discovery of additional reserves.

2.3 Model 3: Reserves growth

This model is based on Pindyck (1978) who argued that there is an inverse relationship between marginal extraction costs and reserves. In this model, a firm has an incentive to explore and develop new reserves in order to drive down marginal extraction costs.

Let R_t be reserves in period t. Additions to reserves, $f(w, S_t)$, depend on effort, w, and cumulative discoveries, S_t, where $f_w > 0$, $f_S < 0$, $S_{t+1} - S_t = f(w, S_t)$ and $S_1 = 0$. Changes in reserves are then $R_{t+1} - R_t = f(w_t, S_t) - q_t$ where $R_1 = 0$. Let the cost of effort be $c(w)$, where $c' > 0$ and $c'' \geq 0$, and costs of extraction be $C(q, R)$ where $C_q > 0$, $C_{qq} \geq 0$, $C_R < 0$, and $C_{qR} < 0$. Thus costs and marginal costs are decreasing in reserves. Profit maximization is

$$\max_{q_t, w_t} \sum_{t=1}^{\infty} \beta^t [p_t q_t - C(q_t, R_t) - c(w_t)]$$

subject to the equations of motion for reserves and cumulative reserve additions. The first order conditions can be written

$$p_t = C_q(q_t, R_t) + (1 + r)^t \lambda_t \tag{2}$$

$$\beta^t c'(w_t) - \gamma_t f_w(w_t, S_t) = \lambda_t f_w(w_t, S_t) \tag{3}$$

[13] The quadratic trend is simply a regression of price on a second degree polynomial of time. This result, like that of Slade, may be spurious as discussed above. Lee *et al.* present a superior method for analyzing the quadratic trend.

$$\lambda_{t+1} - \lambda_t = \beta^t C_R(q_t, R_t) \tag{4}$$

and

$$\gamma_{t+1} - \gamma_t = -(\lambda_t + \gamma_t) f_S(w_t, S_t) \tag{5}$$

where $\lambda_t > 0$ and $\gamma_t < 0$ are the shadow values of reserves and of cumulative additions to reserves at time t. Equation 2 sets the marginal benefit of oil equal to the full marginal cost. Equation 3 sets the marginal cost of effort plus the scarcity cost of effort equal to the marginal benefit of effort in terms of increased reserves. Equations 4 & 5 are the equations of motion for the shadow values.

For a constant price, $\bar{p}$, substituting Equation 2 into Equation 4 shows that

$$r[\bar{p} - C_q(q_t, S_t)] + C_q(q_{t+1}, S_{t+1}) - C_q(q_t, S_t) + (1 + r)C_R(q_t, S_t) = 0. \tag{6}$$

Since the first term in Equation 6 is positive and the last term is negative, the change in marginal extraction costs, $C_q(q_{t+1}, S_{t+1}) - C_q(q_t, S_t)$, can be either positive or negative. In fact, Pindyck shows that if initial reserves are low, it is optimal for the firm to exert effort to find reserves to drive down the marginal extraction cost. This leads to a peak in production and, in equilibrium, a U-shaped price path.

Model 3 is theoretically appealing since the peak is clearly endogenously determined and is not driven by exogenous shifts. However, the empirical evidence of a negative relationship between costs and reserves is not conclusive. Figure 8 shows a strong correlation over time between reserves and production. However, this does not suggest a negative relationship between reserves and costs. Livernois & Uhler (1987) present empirical evidence arguing that aggregate reserves and cost are not negatively correlated and in fact are positively correlated. They claim that this may be due to the fact that lower cost reserves tend to be found first. Thus, even if new discoveries are sufficient to increase reserves, they may not lower costs. They argue for a disaggregated analysis and find evidence for the assumed negative correlation between costs and reserves at a disaggregate level.[14]

Pesaran (1990) uses a similar model for an integrated econometric estimation using data on North Sea production.[15] Despite the impressive integration of theory and econometrics, the rational expectations model generates "highly implausible estimates" of marginal extraction costs.

[14] Their estimates should be interpreted with caution since they likely suffer from simultaneity bias.

[15] Pickering (2002) applies a similar model to find evidence of the "discovery decline phenomenon.

Nonetheless, the author argues that the estimates give a "reasonable degree of support to the theory."

2.4 Model 4: Site development

Models 2 & 3 relied on decreases in marginal extraction costs to drive the peak and U-shaped price path. The peak in Model 4 is driven by increases in aggregate production capacity due to production at newly developed sites, despite the fact that costs are unchanged at all previously developed sites.

Each year in model 4, firms choose how large a site to explore and develop and the production capacity to install in that site. Once capacity is installed, production continues from that site until all oil is exhausted. Assume the density of oil is X so the stock of oil in a site of size s is $X \times s$. Costs of exploring a site of size s, are given by $G(s)$ where $G' > 0$ and $G'' \geq 0$. Convex exploration costs could arise from the fixed number of trained geologists or exploration crews in the short run. Before production can take place at a given site, firms must install production capacity, K, in the site. Costs of installing capacity, F, are assumed to be increasing in the amount of capacity, $F' > 0$, at a nondecreasing rate, $F'' \geq 0$.[16] Convex costs in capacity installation could arise due to a fixed number of experienced drilling crews and trained engineers or due to the declining natural pressure of the reservoir.[17] Finally, extraction costs for any site, $C(q, K)$, depend on the amount extracted as well as the installed capacity with $C_q > 0$, $C_{qq} > 0$, $C_K < 0$, $C_{KK} > 0$, and $C_{qK} < 0$. Thus, pumping costs and marginal pumping costs are increasing in output but decreasing in capacity. The remaining cost assumption, $C_{KK} > 0$, implies that the decrease in costs from adding capacity is smaller at higher levels of capacity, *i.e.,* $d(-C_K)/dK < 0$.[18]

Profit maximization is

$$\max_{q_{it}, K_t, s_t} \sum_{t=1}^{\infty} \beta^t [p_t Q_t - \sum_{i=1}^{t} C(q_{it}, K_i)] - \beta^t F(K_t) - \beta^t G(s_t)$$

[16] For simplicity, assume that capacity costs are independent of the size of the site.

[17] If K is interpreted as effective capacity, then increasing effective capacity by 100 bpd would require a smaller capital investment when capacity is smaller, due to the natural pressure within the reservoir.

[18] This specification of the cost function does not allow an inverse L-shaped marginal cost curve. The allowable function $C(q, K) = cq + (K - q)^{-\alpha}$ defined for $q < K$ approximates an inverse L-shaped marginal cost curve as $\alpha \to \infty$.

subject to

$$
\begin{aligned}
q_{it} &= 0 \qquad \text{if } \; i > t \\
Q_t &= \sum_{i=1}^{t} q_{it} \qquad \forall t \\
\sum_{t=1}^{\infty} q_{it} &\leq X \times s_i \quad \forall i \\
\sum_{i=1}^{\infty} s_i &\leq S \quad \forall i
\end{aligned}
$$

where q_{it} is production from site i at time t, K_t is capacity installed in site t at time t, and s_t is the size of the site explored at time t, and S is the total area available for exploration. Production profits are revenue minus production costs from all sites that have previously been explored and developed. In each period, an additional site is explored and developed (*i.e.,* production capacity is installed in the site) and the final two terms of the objective function capture the costs of development and exploration of a new site.[19] The first constraint prevents extraction from a site before it is explored and developed, and the second constraint defines aggregate supply as the sum of production from all developed sites. The third constraint is the stock constraint for site i, and the final constraint ensures that the size of all the developed area is less than the total area available for development.

The first order conditions can be written:

$$p_t \leq C_q(q_{it}, K_i) + (1+r)^t \lambda_i \qquad \forall t, \;\; \forall i < t \tag{7}$$

$$\sum_{t=i}^{\infty} \beta^t [-C_K(q_{it}, K_i)] = \beta^i F'(K_i) \qquad \forall i \tag{8}$$

$$G'(s_i) + (1+r)^i \gamma = (1+r)^i \lambda_i \times X \qquad \forall i \tag{9}$$

where λ_i is the shadow value of oil at site i, and γ is the shadow value of area for development. The first condition says that the price equals the full marginal cost at each site with positive production. The second condition implies that the present value of the sum of cost reductions from an additional unit of capital must equal the present value of the cost of the additional unit of capital. The third condition says that the marginal cost of exploration plus the scarcity cost

[19] Decreasing returns in exploration ensure the entire available area is not explored and developed in the first period.

of the additional explored area must equal the benefit from an additional barrel of oil times the density of oil.

To illustrate the solution to the model, first let price be constant, *i.e.,* $p_t = \bar{p}$ for every t, and let $S = \infty$. With unlimited area to develop, the shadow value, γ, would be zero, and the exploration and development problems would be stationary. This stationarity implies that it is optimal to explore the same size site in every period and to install the same capacity in every period, *i.e.*, , $s_t = s_1$ and $K_t = K_1$ for every t. Equation 9 then implies that $\lambda_t = (1+r)\lambda_{t+1}$ so that the shadow values of oil are declining at sites that are developed later.[20] Now turn to production from site 1. Since capacity is fixed after installation, the marginal cost curve is fixed over time. However since the scarcity cost of oil at site i, $(1+r)^t\lambda_i$, is growing over time, production from site i is decreasing, and oil at the site is eventually exhausted. Figure 9 illustrates this declining production over time from site i as the full marginal cost curve shifts upward. Now consider aggregate production over time. In period 1, only one site has been explored and developed, so aggregate production is just q_{11}. In period 2, two sites have been explored and developed, so production is $q_{12} + q_{22}$. By stationarity, $q_{11} = q_{22}$ so aggregate production increases in the second period even though production at site 1 has decreased. In the third period, production begins at site 3 so aggregate production increases despite decreasing production at the two sites that were previously explored and developed. As more sites are developed, aggregate production increases further until site 1 is exhausted. After this, new production in each period exactly offsets the declines in production at all the other sites, and aggregate production would be at the steady-state level, $X \times s_1$.

Since there is limited area to develop, *i.e.,* S is finite, the exploration and development problems are not stationary. Consider s_i^*, the optimal site size in period i. In period $i+1$, suppose the firm were to explore the same size site. As argued above, the shadow value of oil, $\lambda_i(s_i^*)$, which is decreasing in s_i^*, must be smaller at the later site such that $\lambda_i(s_i^*) = (1+r)\lambda_{i+1}(s_i^*)$. But this implies that $G'(s_i^*) + (1+r)^{i+1}\gamma > G'(s_i^*) + (1+r)^i\gamma = (1+r)^i\lambda_i(s_i^*) \times X = (1+r)^{i+1}\lambda_{i+1}(s_i^*) \times X$. Comparing this with equation 9 shows that in period $i+1$ the marginal cost of exploration would

[20] Intuitively, a marginal increment of oil to a site that is developed later is worth less than the marginal increment to a site that is developed sooner.

be greater than the marginal benefit of exploration if the firm explored the same size site. Thus the optimal site size is declining over time, *i.e.*, $s_i > s_{i+1}$. This argument is illustrated graphically in Figure 10 which shows the marginal benefit of exploration $(1+r)^i \lambda_i(s)X$ and the marginal cost of exploration $G'(s) + (1+r)^i \gamma$ both conditional on site size s for periods i and $i+1$. If s is the same in two periods, then $\lambda_i(s) = (1+r)\lambda_{i+1}(s)$, and the marginal benefit of exploration is equal in periods i and $i+1$. However, the marginal cost of exploration increases due to the scarcity of sites as illustrated.

Now turn to capacity installation. Let K_i^* be the optimal capacity to be installed at site i. The firm could install the same capacity in the next site in the following period. Let $q^*_{(i+1)t}$ be the optimal production path at site $i+1$ conditional on installing capacity K_i^*. This implies that:

$$\sum_{t=i+1}^{\infty} \beta^{t-i-1}[-C_K(q^*_{(i+1)t}, K_i^*)] < \sum_{t=i+1}^{\infty} \beta^{t-i-1}[-C_K(q_{i(t-1)}, K_i^*)]$$

$$= \sum_{t=i}^{\infty} \beta^{t-i}[-C_K(q_{it}, K_i^*)] = F'(K_i^*). \tag{10}$$

The first inequality holds because site $i+1$ is smaller than site i. This implies that optimal extraction from site $i+1$ is smaller than extraction from site i in the preceding period, *i.e.*, $q^*_{(i+1)t} \leq q_{i(t-1)}$ for all t with strict inequality when $q_{i(t-1)} > 0$. Since $C_{qK} < 0$, it follows that $-C_K(q^*_{(i+1)t}, K_i^*) < -C_K(q_{i(t-1)}, K_i^*)$ for $q_{i(t-1)} > 0$. The second equality in Equation 10 follows by rearranging the indices, and the final equality follows from equation 8. But the inequality in Equation 10 implies that the marginal benefit of installing capacity K_i^* at site $i+1$ (the reduction in pumping costs) is less than the marginal capacity cost of K_i^*. Thus, optimal capacity is smaller at the smaller site, i.e., $K_i > K_{i+1}$ for every i.

Characterization of the Optimal Supply: With a stationary output price, limited total area for exploration, and stationary costs of production, exploration and development, the following hold:

- **(i) Declining production at each site:** $q_{it} > q_{i(t+1)}$ if $q_{it} > 0$.
- **(ii) Exhaustion at each site:** $\exists\, T(i) > i$ such that $q_{it} = 0$ for every $t > T(i)$.
- **(iii) Decreasing capacity:** $K_i > K_{i+1}$.
- **(iv) Decreasing site size:** $s_i > s_{i+1}$.

(v) Increasing and decreasing aggregate production: Production increases if additional production at the newly developed site offsets production declines at all the previously developed sites. Eventually production declines as newly developed sites become smaller.

Optimal production, exploration, and capacity installation can be simulated using an algorithm of nested loops. For any given γ, s_1, K_1, and λ_1, the extraction path q_{1t} is determined by equation 7. If total extraction from site 1 is greater (less) than $s_1 \times X$, then the marginal user cost, λ_1, must have been too small (large). Using this adjustment rule, the optimal shadow value, conditional on γ, s_1, and K_1, can be calculated by looping. Next equation 8 is used to determine whether too much or too little capacity has been installed in this site. This adjustment rule allows the optimal capacity to be calculated by looping where the optimal shadow value is calculated during each iteration. Once K_1 and λ_1 are computed optimally, equation 9 can be used to determine whether the given s_1 is too large or too small. This adjustment rule can be used, with nested loops for K_1 and λ_1, to compute the optimal site size, s_1, for a given γ. To determine whether γ is too high or too low, the optimal site size must be computed for each site. If the sum of the sites is greater (less) than S, then γ was too low (high). This adjustment rule, with nested loops for s_i, K_i, and λ_i, allows the optimal γ to be computed.

Figure 11 illustrates optimal production in the baseline model for the first seven sites and for the aggregate.[21] In the first year, production is only from the first site so aggregate production is q_{11}. In the second year, production starts from the second site while production declines slightly from the first site. Thus aggregate production increases in the second year. Aggregate production continues to increase until year 13 as new production offsets declines in production at existing sites. After the peak, new production cannot offset declines in production at existing sites. The first site is exhausted by year 17 and thereafter at least one site is exhausted every year. The last site is developed in year 51, and all sites are exhausted by year 53.

Figure 12 shows a schematic of Hubbert's curve taken from a primer on peak oil (Energy Bulletin 2006). The schematic, not based on an underlying model, illustrates the observed (hypothesized?) relationship between aggregate production and production at individual wells.

[21] Parameters of the baseline simulation are $\bar{p} = \$50$, $C(q, K) = q^2/K$, $G(s) = s^2/2$, $F(K) = K^2/2$, $S = 20,000$, $X = 100$ and $r = 0.05$.

Model 4 could be parameterized to match this schematic, thus showing economic assumptions which generate such a schematic.

Figure 13 shows the relationship between "reserves" in Model 4 and production where reserves are defined in the model as the unextracted oil in all the developed sites.[22] Here reserves peak before production does. Figure 8 shows that U.S. reserves and production peaked roughly at the same time.

Figures 14-16 illustrate the comparative dynamics of the model when the discount rate changes. Figure 14 shows that both the peak and exhaustion occur earlier at the higher interest rate of 0.07%. This change has an analog in the standard Hotelling model in which production declines faster and exhaustion occurs earlier when firms are more impatient. Figure 15 shows the falling site size over time and Figure 16 illustrates the falling capacity size over time. Both site size and capacity are larger initially and decline faster at a higher interest rate.

The discussion has focused on the supply side of the model. Similar to Models 2 & 3, the equilibrium price path in Model 4 will be U-shaped if demand is not perfectly elastic. Furthermore, the production peak and lowest price would be coincident if demand were static, but, with increasing demand, the production peak will occur after the lowest price.

3 Policy analysis

These models can be used to analyze a variety of policy proposals, the simplest of which is an energy tax.[23] Such a tax could improve efficiency if it corrected some market failure. Possible market failures include external environmental costs, external national security costs, and incomplete futures markets. Caplin & Leahy (2004) point out that the competitive equilibrium is the most impatient Pareto optimal intertemporal allocation. Thus a social planner might prefer a more patient optimum. Finally, there may be macroeconomic adjustment costs that are not captured by the competitive equilibrium. Normative analysis would require more complete modeling of the market failures, so here I simply assess the positive effects of an energy tax. Presumably the goals of the tax are: (i) to delay the peak, (ii) to delay exhaustion of the resource, and (iii) to reduce

[22] To be precise, reserve additions in year t are $s_t \times X$, and reserve reductions are Q_t.

[23] Kaufmann (Econoblog 2005) proposes a "large" energy tax phased in over 20 years.

volatility in oil production.

Figures 17-20 present simulation results from adding a \$10 energy tax at various times in Model 4.[24] The tax is announced in year zero and is fully anticipated. Figure 17 illustrates a \$10 tax adopted in year zero. This figure shows that an energy tax can be effective in attaining the stated goals: the peak and exhaustion are delayed and production volatility (amplitude of the peak) is reduced.

However, the effectiveness of the announced energy tax depends crucially on its timing. Figure 18 illustrates the same \$10 tax instead adopted in year 5. This figure shows that the energy tax still attains the first two goals since the peak and exhaustion are delayed. However, goal (iii) is more difficult to evaluate. The tax does reduce the amplitude of the peak but induces a sharp decrease in production in year 5 when the tax is adopted. Thus the tax reduces the long-term volatility at the expense of some additional short-term volatility.

Figure 18 also hints at some other surprising consequences that can result from an energy tax. Note that the tax—here fully anticipated by the producers—affects production both before and after its adoption. Namely, the tax increases production before its adoption (when the after-tax price is relatively high) and decreases production after adoption (when the after-tax price is relatively low). This can have surprising effects. When the tax is scheduled to be adopted shortly before the peak, as in Figure 19, the tax can actually cause the peak to occur earlier (here just before the tax is adopted). Although this tax does delay exhaustion of the resource, note the effects on production volatility: the peak is much greater than in the baseline and production drops dramatically in the adoption year. Thus both short- and long-term volatility increase here.

The tax imposed after the peak, illustrated in Figure 20, is even less effective in attaining the stated goals. Although the tax does delay the peak, the resource is exhausted earlier and the production volatility again increases in both the short and long term.

Two important details about the energy taxes analyzed thus far are that they are not phased in and that they are completely anticipated. Clearly, a phased-in tax avoid inducing additional short-term volatility. Figure 21 illustrates an anticipated tax that is phased in over

[24]Since demand in the simulation is perfectly elastic, the tax is borne entirely by the producers.

ten years beginning in year 5. As expected there is no additional short-term volatility and long-term volatility is also reduced. Note, however, that as with the taxes that are not phased in, the anticipated future tax causes current production to be higher. With this tax, production is maintained above the baseline production for five years after the tax is adopted and the peak is hastened.

Figure 22 shows the effects of an unannounced tax in year 5. Here production is basically unchanged in the adoption year. Although production decreases at each existing site, new production in the adoption year offsets these decreases and aggregate production increases slightly. Thus there is no additional short-term volatility. Since the peak and exhaustion are delayed and there is no additional volatility, this tax quite effective at attaining the policy goals. Note however that an unannounced tax introduced at or after the peak, could have quite different effects. In particular, it could introduce short-term volatility.

Figure 23 shows an unannounced tax phased in from years 5 to 15. Since the tax is phased in, short-term volatility does not increase. Furthermore exhaustion is delayed and long-term volatility is reduced. However, firms increase production above baseline production during the early years of the tax in response to the higher relative prices. This hastens the peak instead of delaying it.

In addition to energy taxes, other policies, many of which serve to reduce future demand for oil, have been proposed. For example, extensive investment in carbon fiber cars would reduce future demand for oil. Are such demand reduction policies amenable to this analysis?

In short, yes. Suppose investing in carbon fiber cars would reduce demand in ten years. This policy would affect the oil market exactly like an announced tax increase in ten years. The investment would cause firms to produce more oil now at higher relative prices and less in the future at lower relative prices. As with the announced tax increase, this could hasten the peak and/or hasten exhaustion of the oil.

4 Conclusion

The four models show that a peak in production is not evidence of market failure but rather that a peak in production could well arise from efficient intertemporal optimization. The four models isolate four possible causes of the increase in production: increasing demand, cost reductions through technological change, cost reductions through exploration, and increasing production from additional site exploration and development. In each model, the underlying scarcity of the resource ultimately leads to a decline in production.

Models 2-4 generate U-shaped price paths in equilibrium. While there is some econometric evidence for a U-shaped price path, the evidence is inconclusive. However, these models can generate production peaks even if price is constant. Thus the conditions under which the models generate production peaks are quite general.

The models do not explicitly address possible market failures, *e.g.,* environmental costs, excessive impatience, or macroeconomic adjustment costs and thus cannot analyze the efficiency of policies. This is an important area for future research. However, the models can illustrate the positive effects of policies. In particular, the models show that energy taxes can effectively attain policy goals such as delaying the peak, delaying exhaustion, and reducing production volatility.

These positive effects depend crucially on whether the policies are anticipated and/or phased in. Simulations show that energy taxes that are anticipated or phased in cause firms to increase production immediately to benefit from an after-tax price that is relatively higher. This can cause a variety of unintended consequences, such as hastening the peak, hastening exhaustion, and/or increasing production volatility in the short and long term. Energy taxes that are unanticipated do not induce firms to increase production prior to the tax. However, they can increase production volatility through a drop in production when the tax is adopted, especially if adoption occurs near the peak. Phased-in taxes can be effective in eliminating sharp drops in production. However, the phase-in period is a time of relatively high after-tax prices and thus can lead to an increase in production and can hasten peaking. These unintended consequences illustrate the importance of careful economic modeling in the evaluation of various policies.

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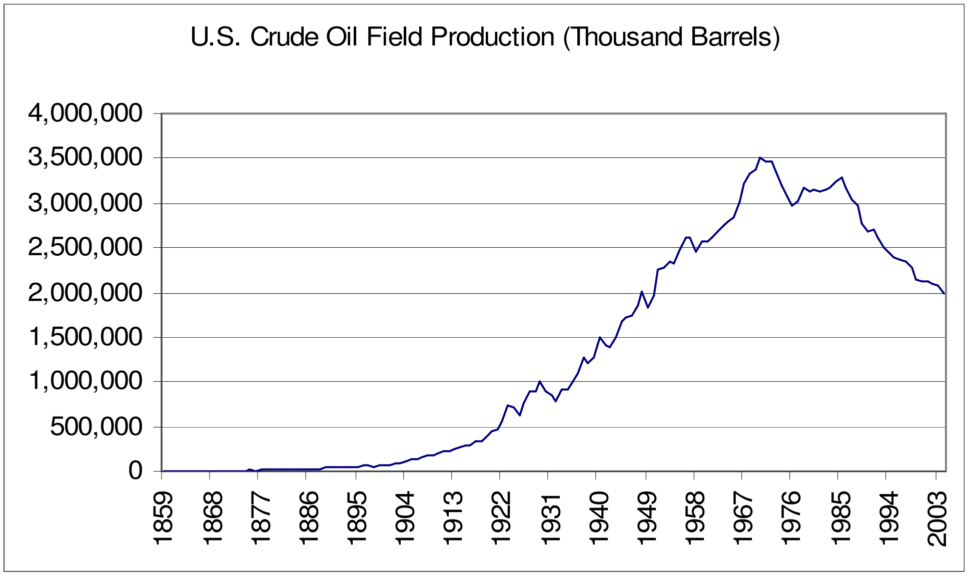

Figure 1: Annual U.S. crude oil production. Source: EIA.

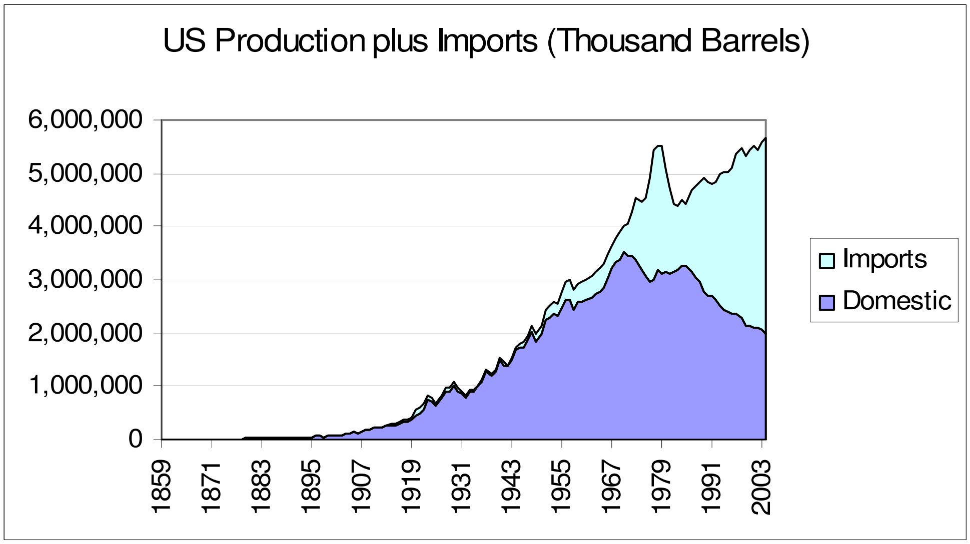

Figure 2: Annual U.S. crude oil production plus imports. Source: EIA.

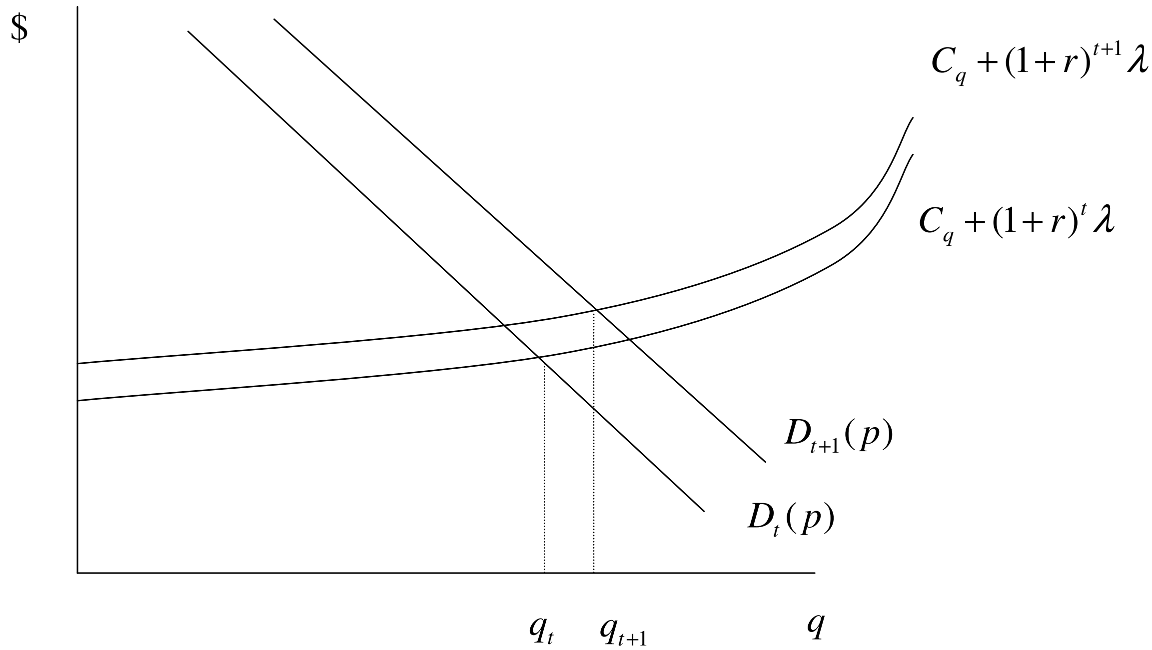

Figure 3: Illustration of a production increase in Model 1, the demand shift model.

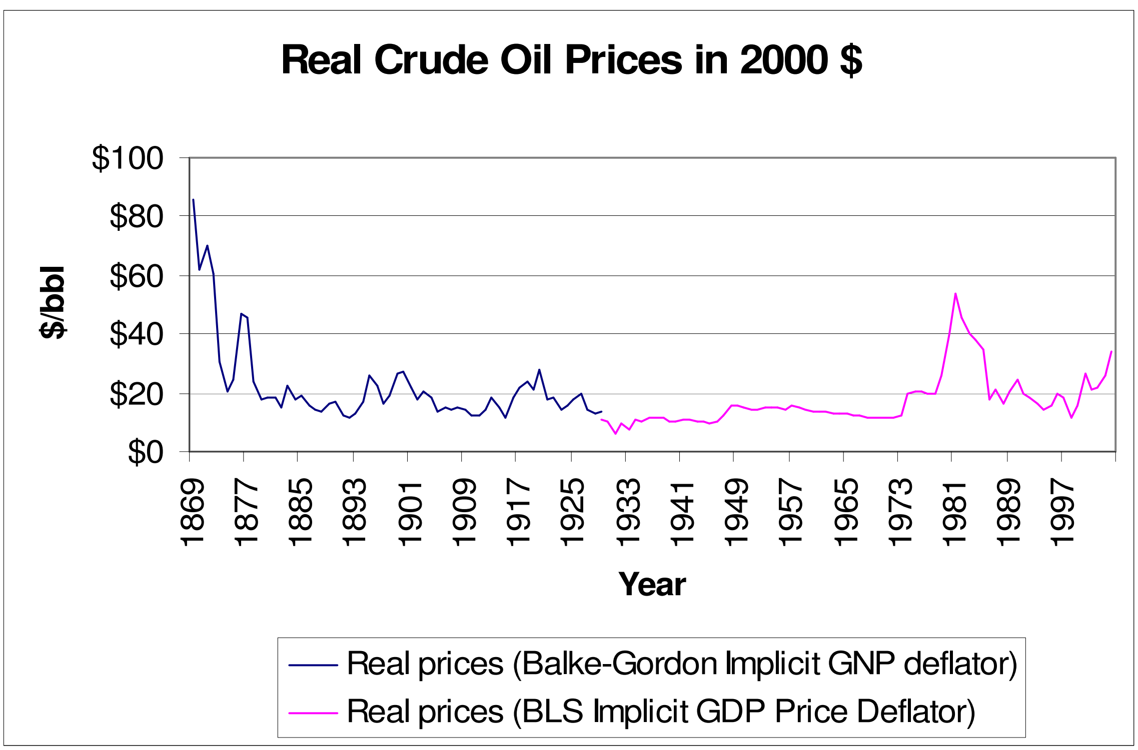

Figure 4: U.S. domestic first purchase crude oil prices in 2000$. Sources: EIA, Manthy, BLS.

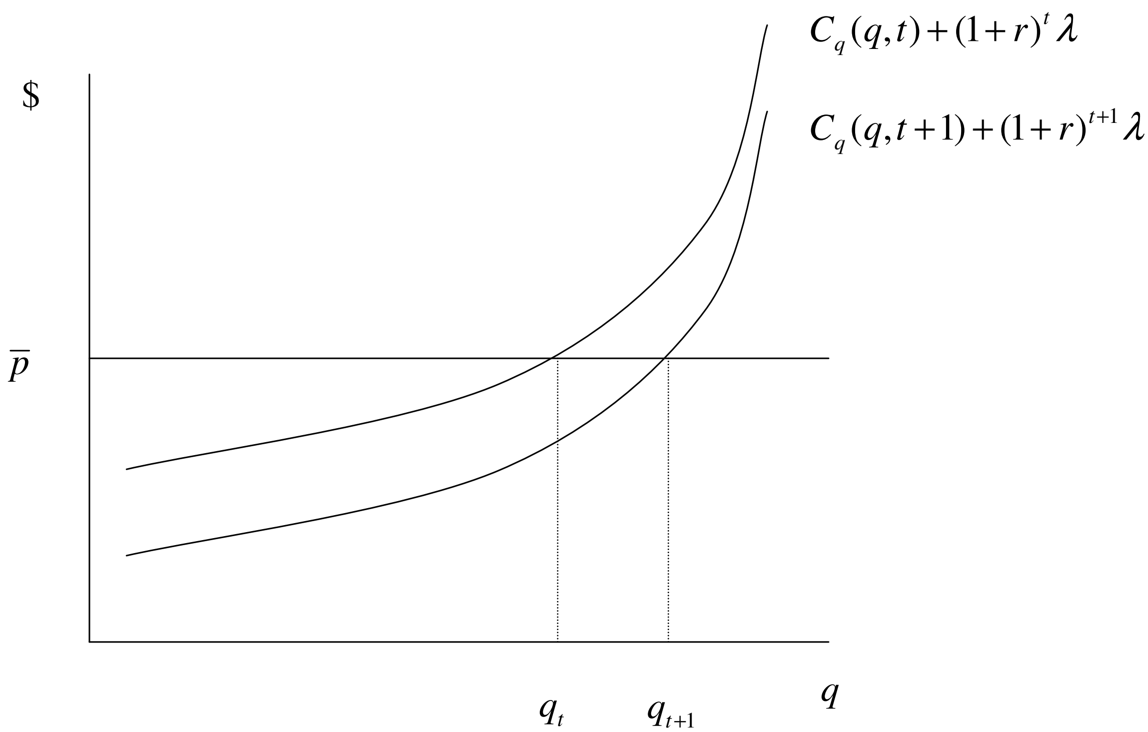

Figure 5: Illustration of increasing production in Model 2, the technological change model.

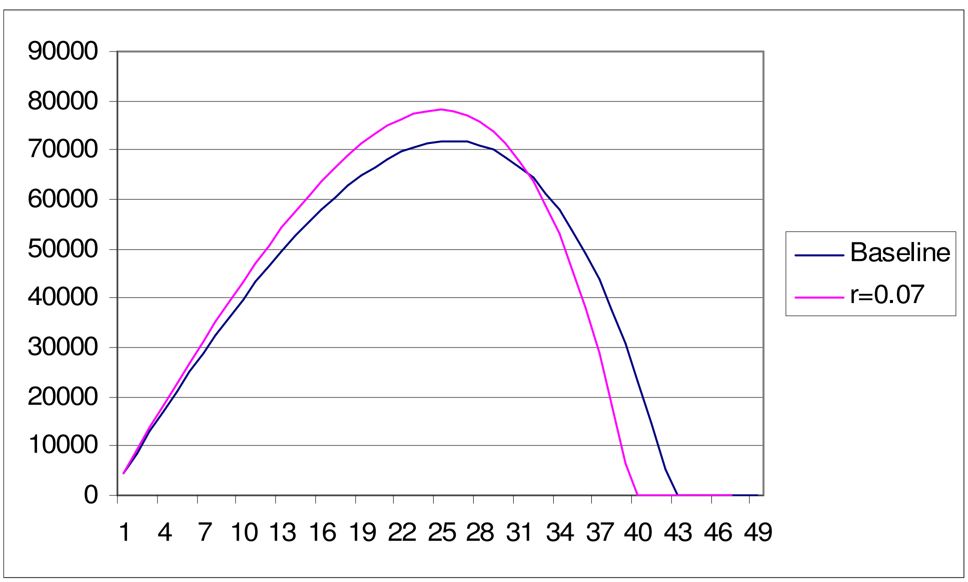

Figure 6: Simulation of a production peak in Model 2, the technological change model.

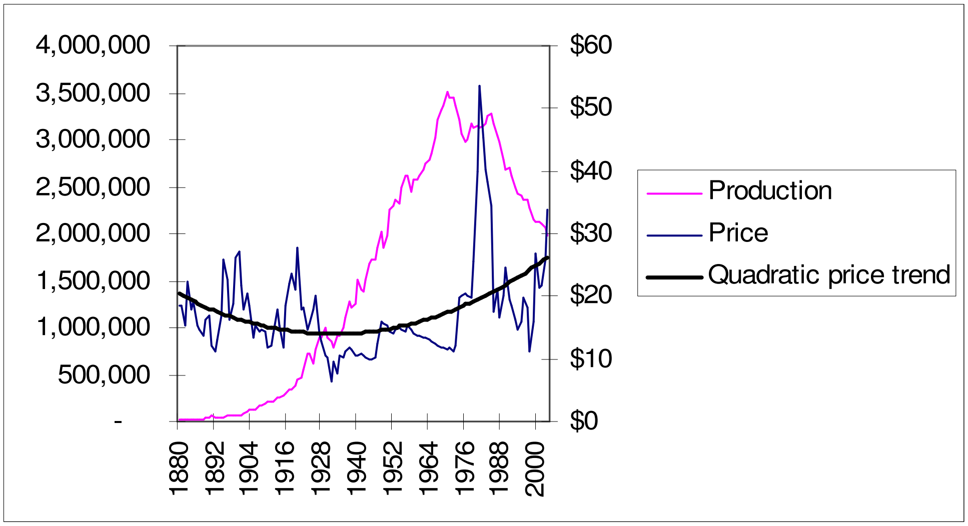

Figure 7: Annual U.S. production and domestic first purchase price. Source: EIA.

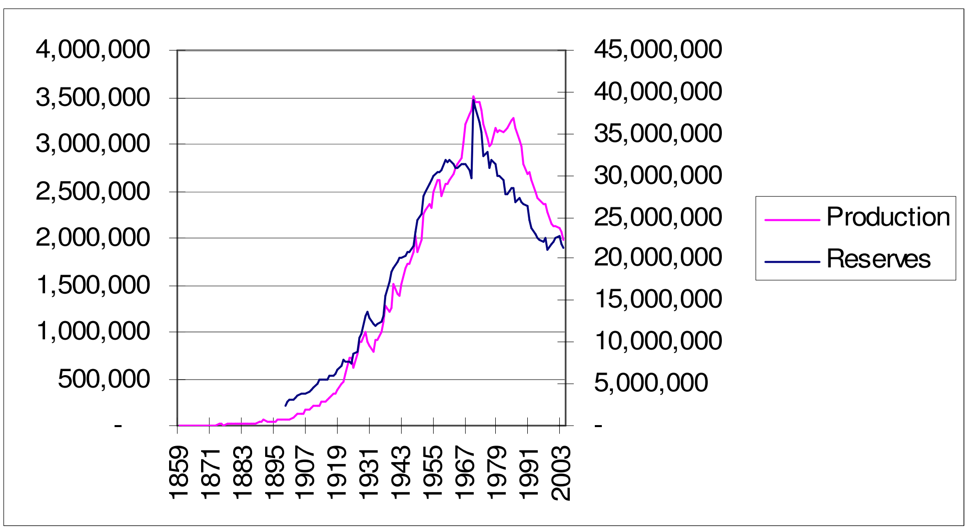

Figure 8: Annual U.S. crude oil production and reserves. Source: EIA.

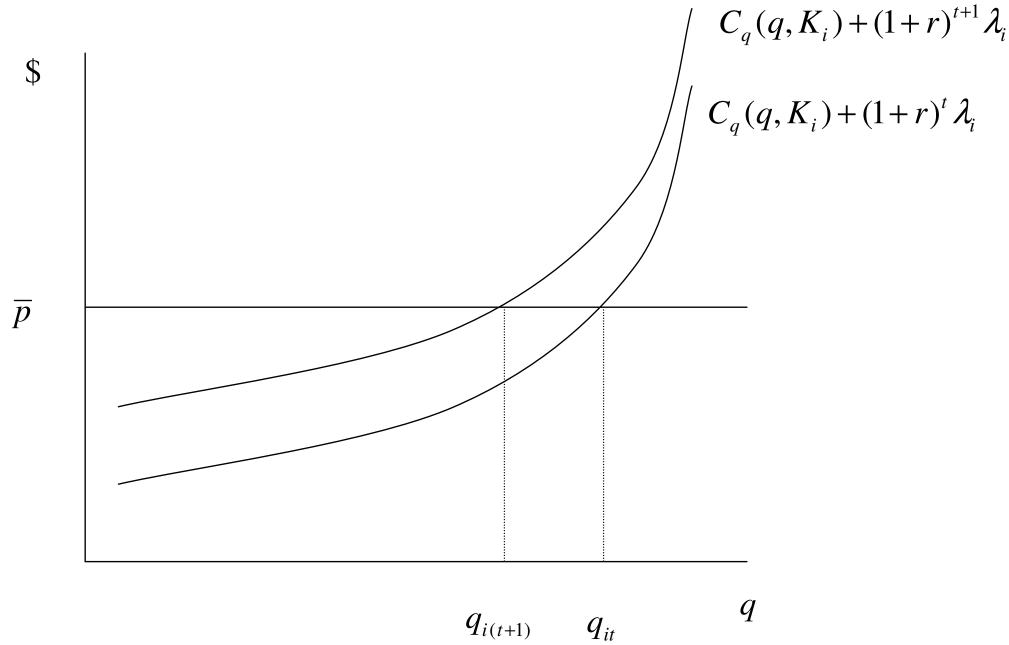

Figure 9: Graphical demonstration of declining production at a site for Model 4, the site development model.

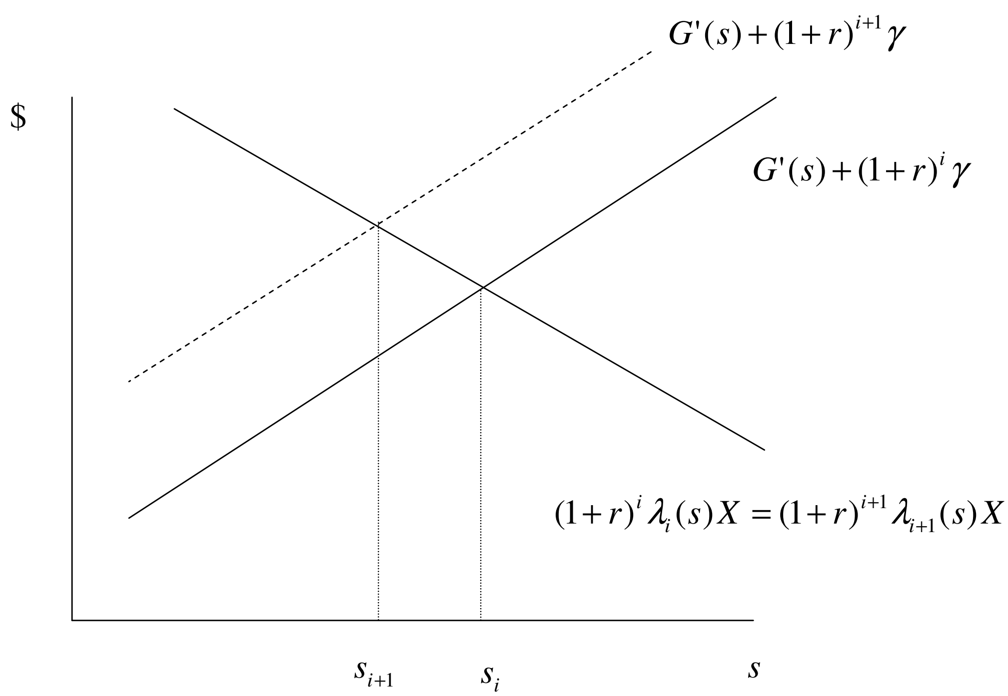

Figure 10: Graphical demonstration of declining site size for Model 4, the site development model.

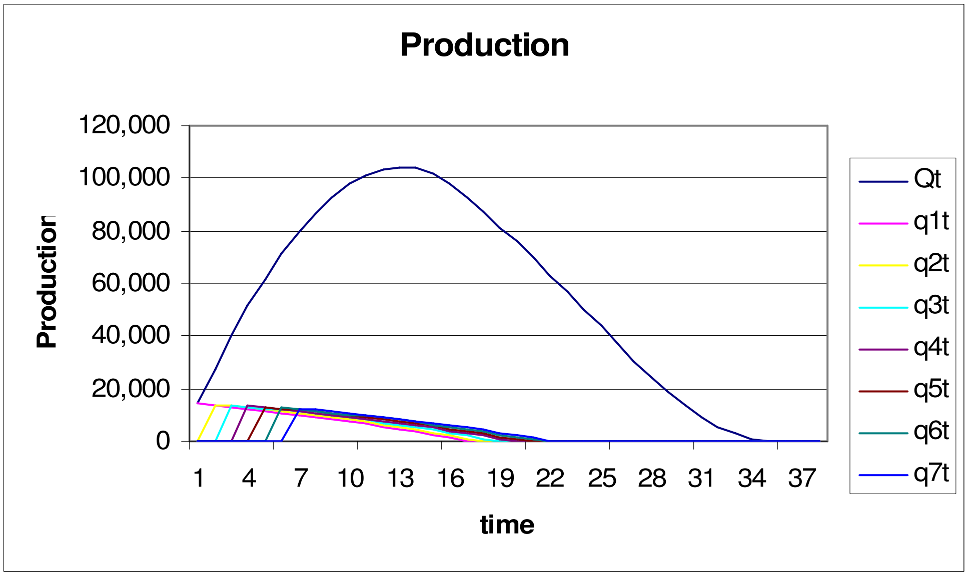

Figure 11: Simulation of aggregate production and production from individual sites in Model 4, the site development model.

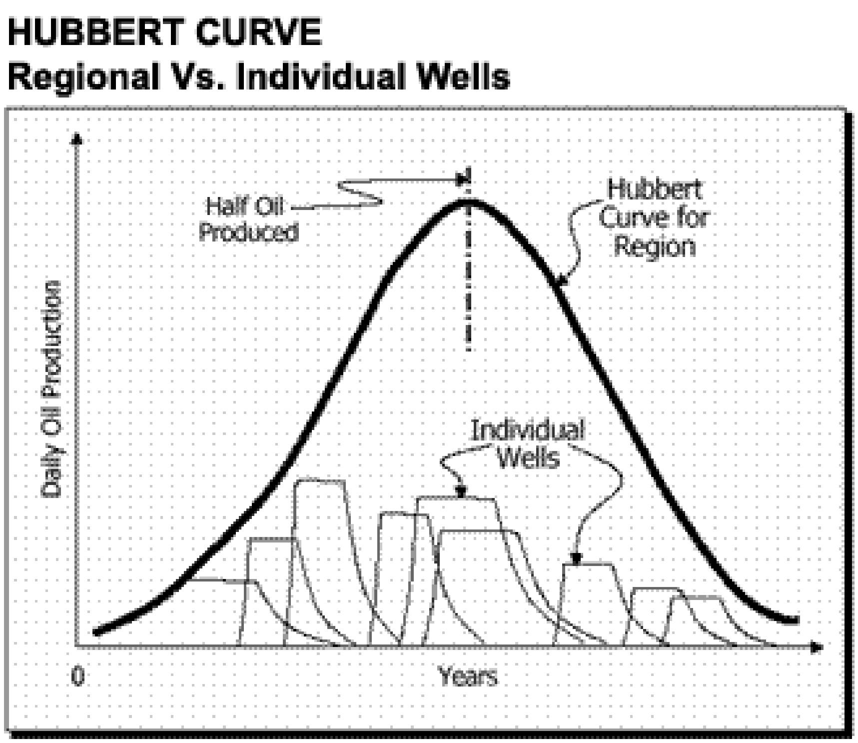

Figure 12: Schematic of Hubbert's curve for a region. Source: Peak oil primer from www.energybulletin.net/primer.php.

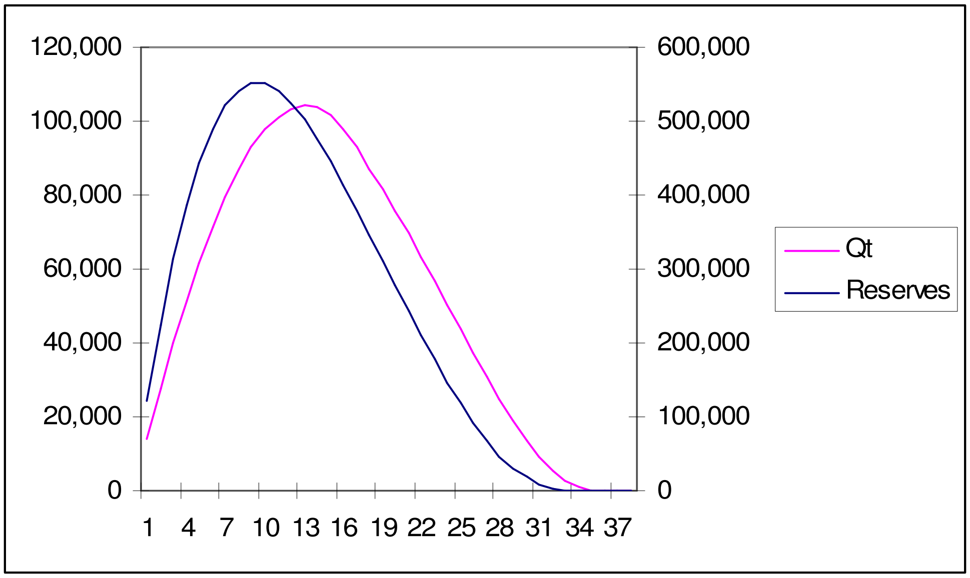

Figure 13: Simulation of aggregate production and reserves in Model 4, the site development model.

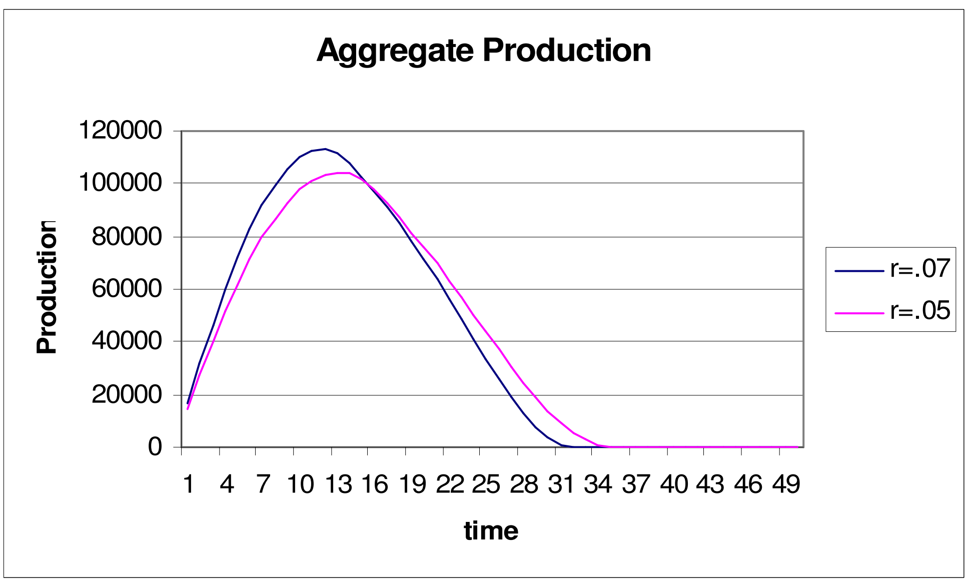

Figure 14: Comparison of aggregate production with different discount rates in Model 4, the site development model.

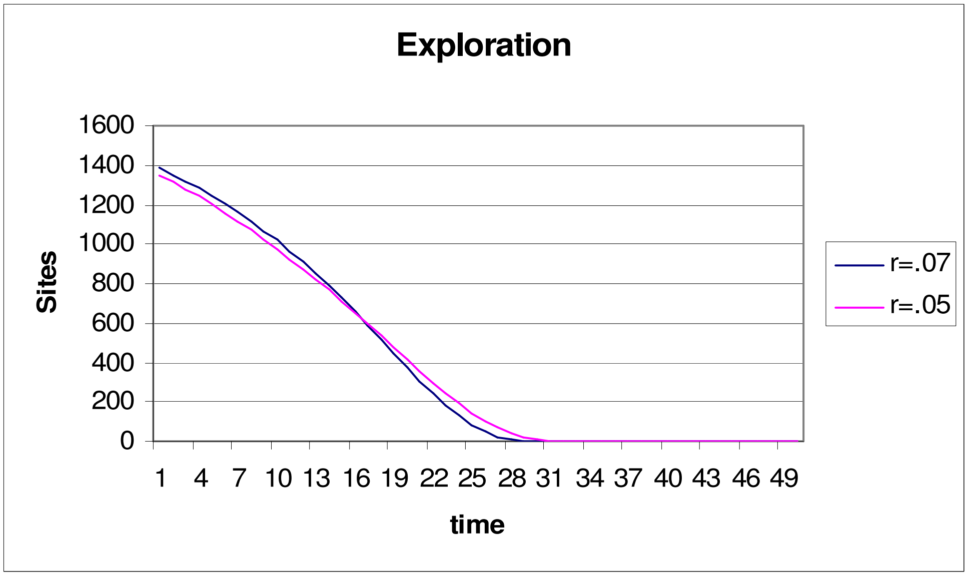

Figure 15: Comparison of site exploration with different discount rates in Model 4, the site development model.

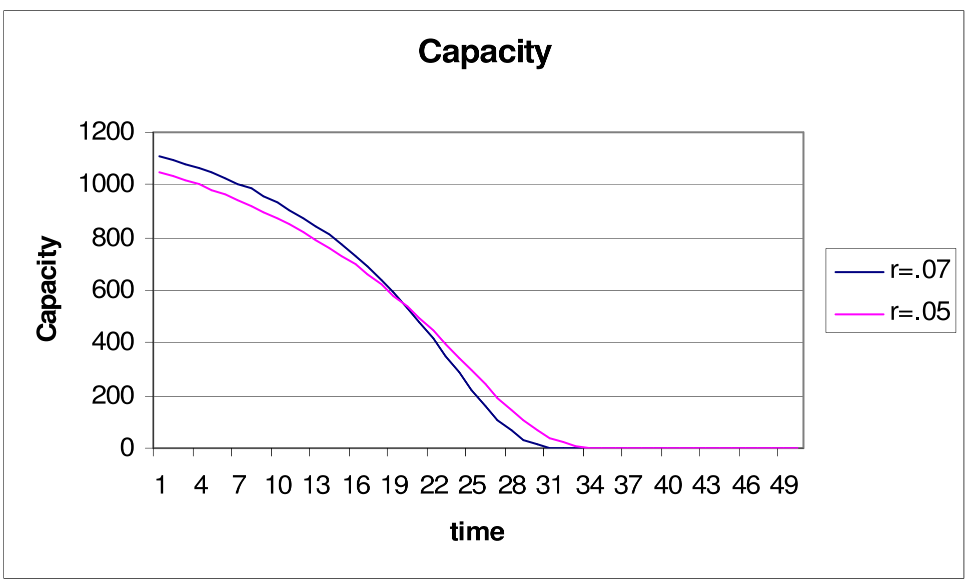

Figure 16: Comparison of capacity installation with different discount rates in Model 4, the site development model.

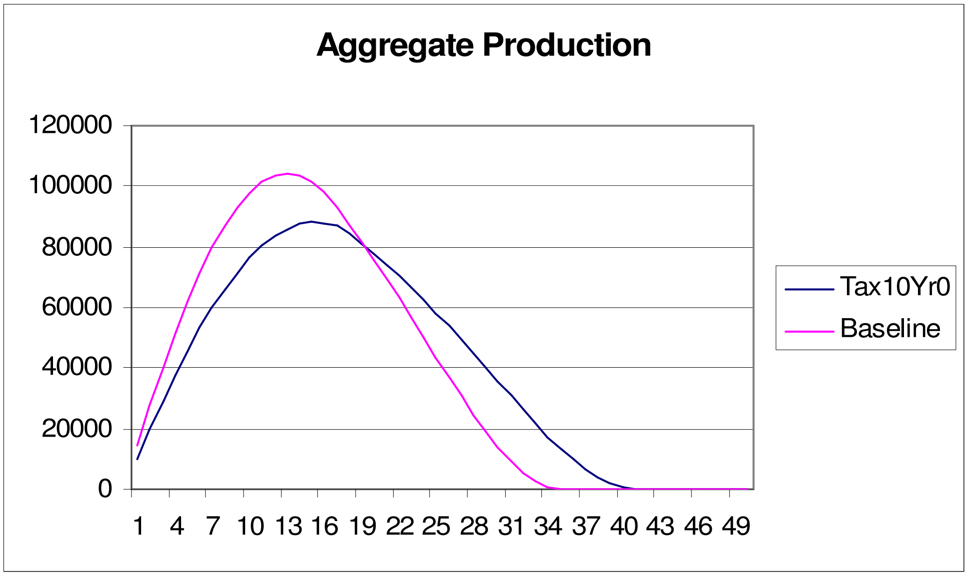

Figure 17: Simulation of an anticipated tax increase in year 0 for Model 4.

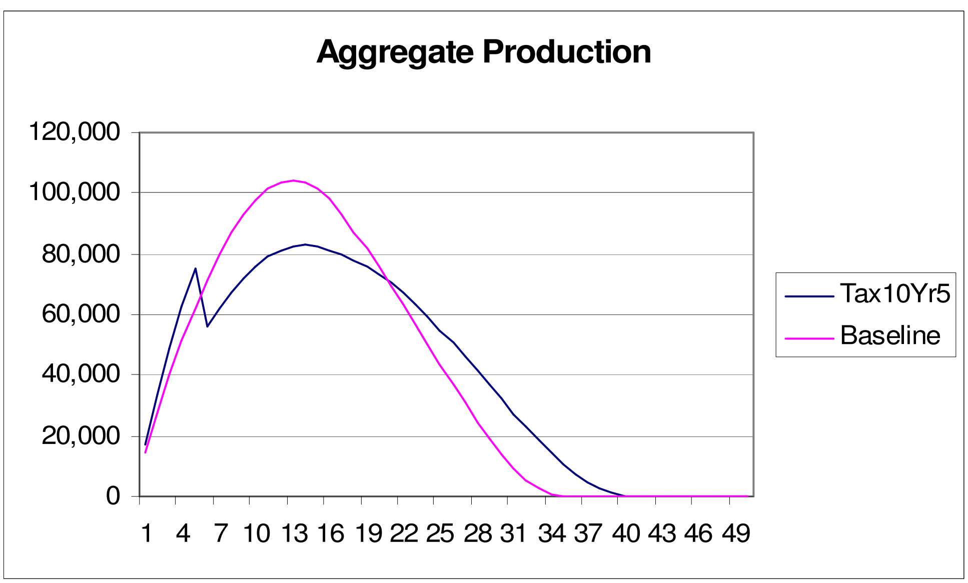

Figure 18: Simulation of an anticipated tax increase in year 5 for Model 4.

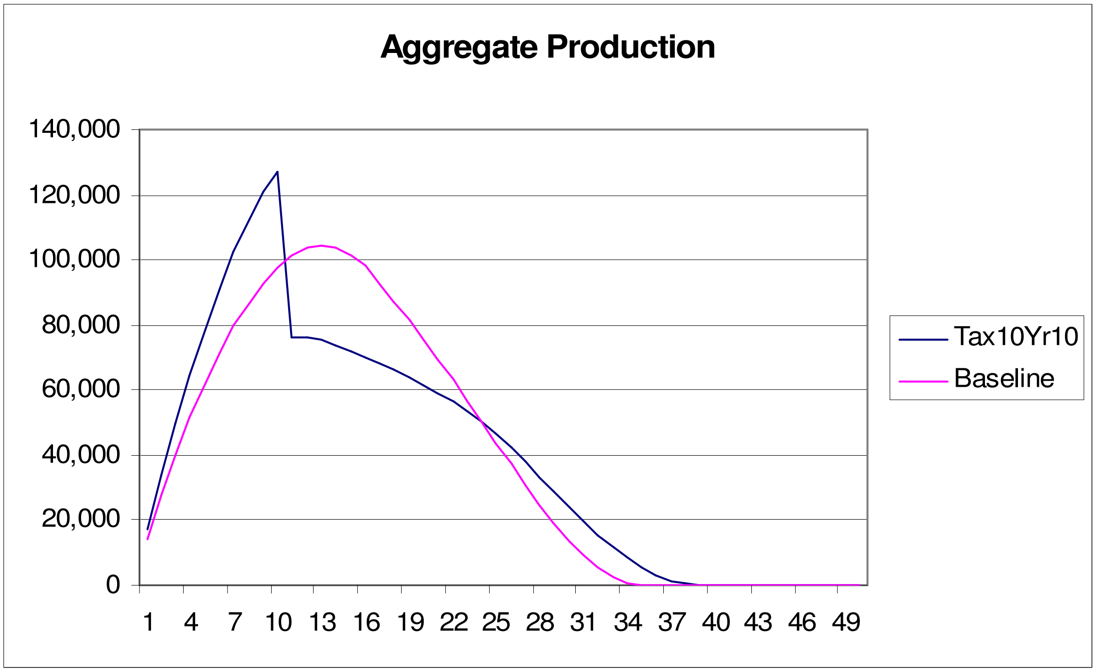

Figure 19: Simulation of an anticipated tax increase in year 10 for Model 4.

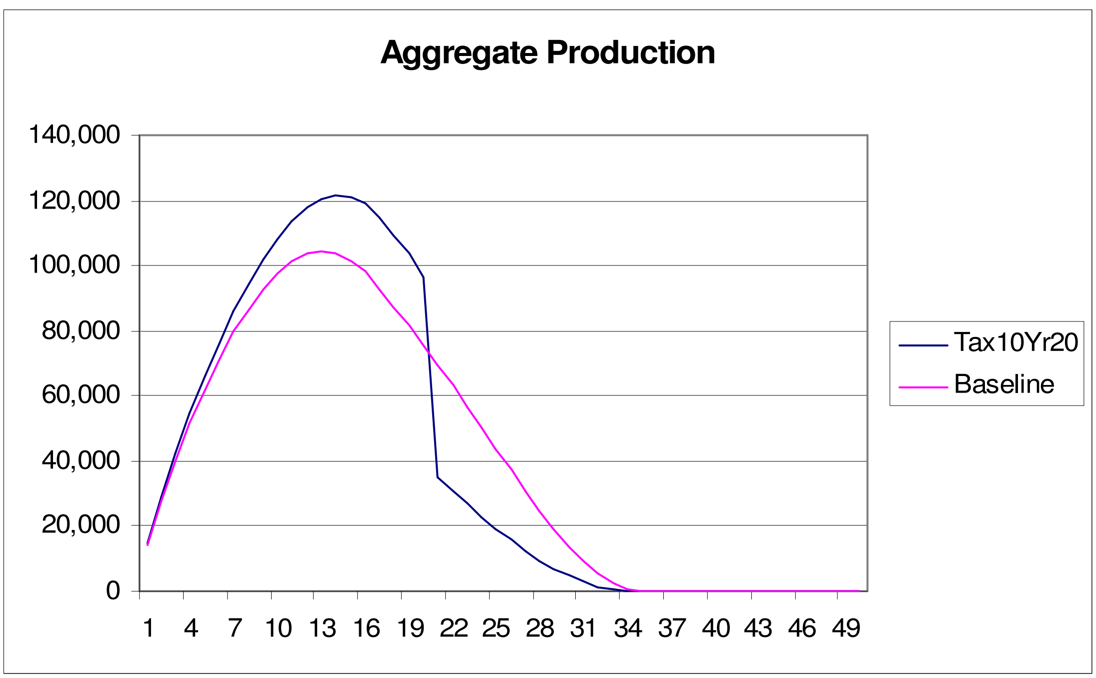

Figure 20: Simulation of an anticipated tax increase in year 20 for Model 4.

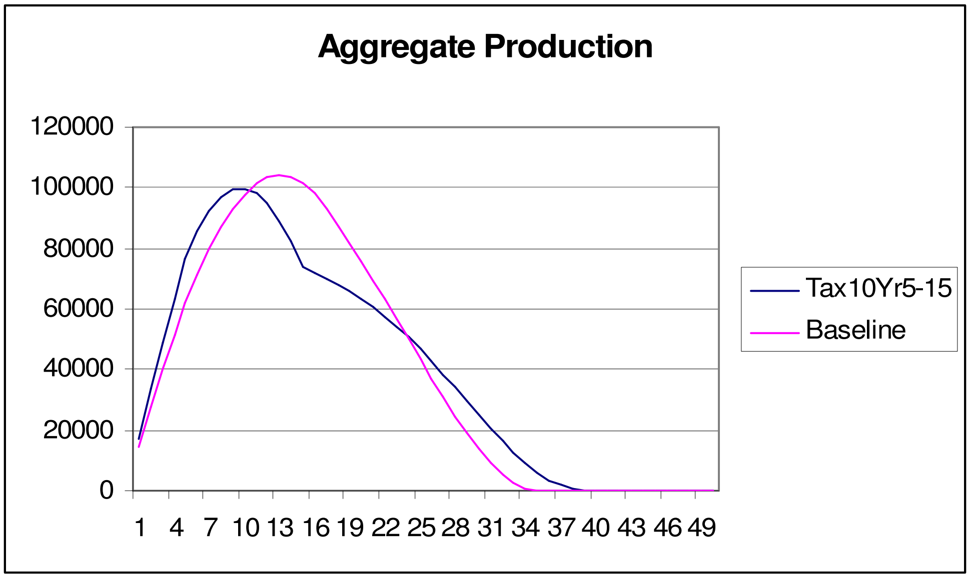

Figure 21: Simulation of an anticipated tax increase phased in between years 5 & 15 for Model 4.

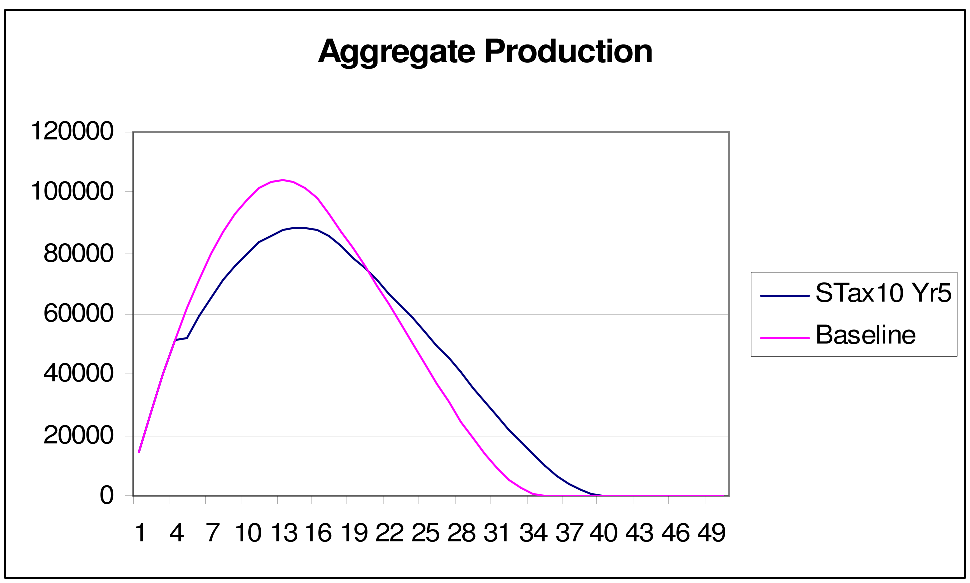

Figure 22: Simulation of an unannounced tax increase in year 5 for Model 4.

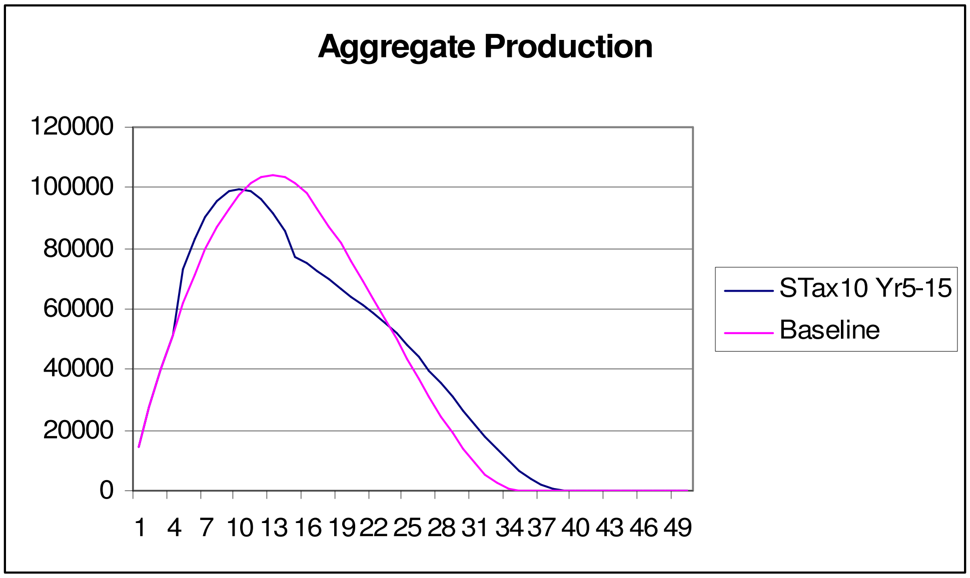

Figure 23: Simulation of an unannounced tax increase phased in over years 5-15 for Model 4.